Z_2 topological phase in quantum antiferromagnets

Masaki Oshikawa
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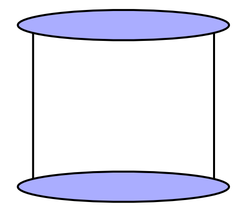
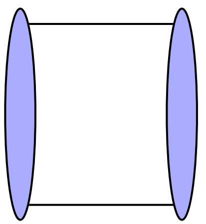
RVB spin liquid

4 spins on a square:

Groundstate is exactly $\frac{1}{\sqrt{2}}\Big($  $+$  $\Big)$

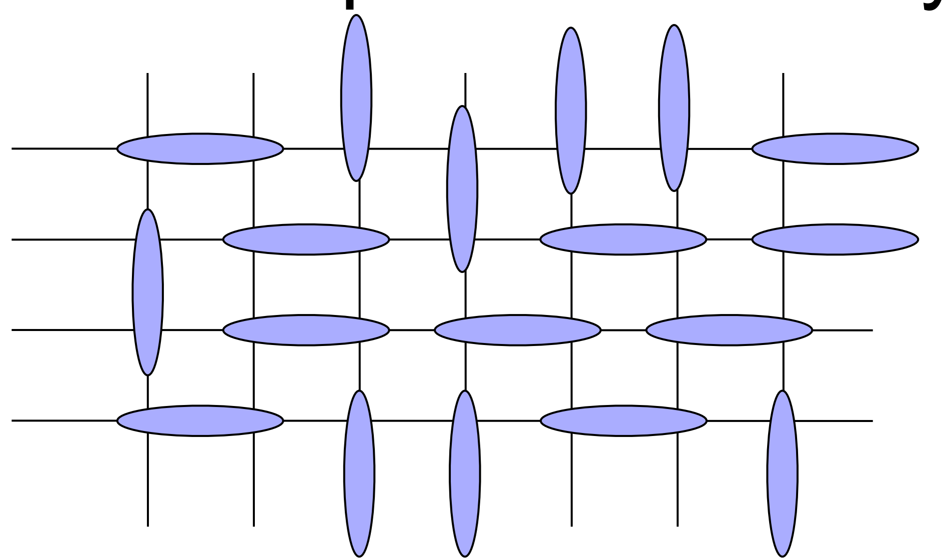
 $= \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

singlet pair

a.k.a. “valence bond”

So, the groundstate on the square *lattice* may be the RVB state:

$$\sum_{\text{all the valence-bond patterns}}$$



?

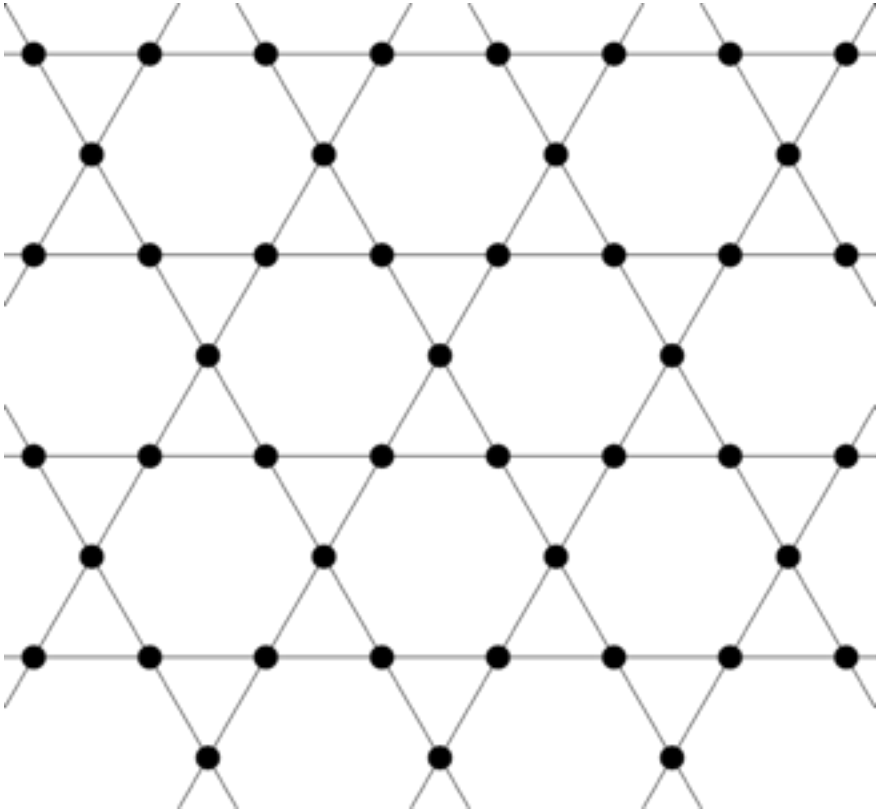
Is it true?

The RVB state was expected to be disordered state without any order – “RVB spin liquid”

However, contrary to the initial motivation, the groundstate of the square-lattice Heisenberg model turned out to have Neel order

Frustration might favor the RVB spin liquid.
However, no case was established
in Heisenberg model on various frustrated lattice
(triangular etc.)

S=1/2 Kagome Lattice HAF



prototypical frustrated magnet

Various proposals

Magnetic order?
Gapless? Gapped?
Valence-Bond Crystal?
Spin liquid?

Spin-Peierls and spin-liquid phases of Kagomé quantum antiferromagnets

J. B. Marston and C. Zeng
Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York 14853-2501

[1991]

PHYSICAL REVIEW B **68**, 214415 (2003)

Physics of low-energy singlet states of the Kagome lattice quantum Heisenberg antiferromagnet

P. Nikolic and T. Senthil
Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, Massachusetts 02139, USA
(Received 22 May 2003; published 12 December 2003)

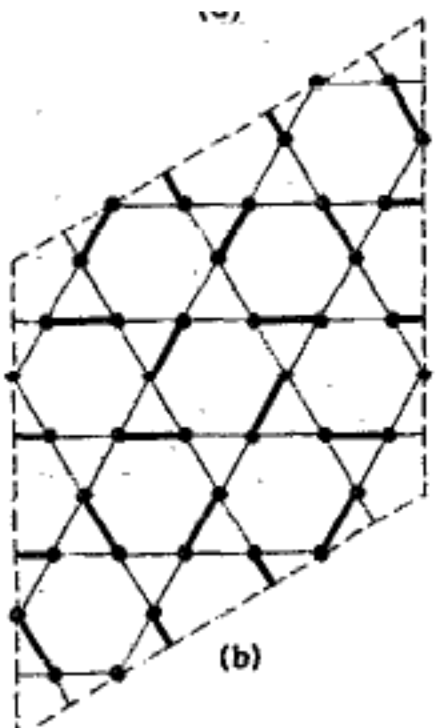


FIG. 1. Unit cells with the maximum fraction of perfect hexagons where the thick lines denote dimers. (a) 18-site unit cell with perfect hexagons that form a oblique lattice. (b) 36-site unit cell with perfect hexagons that form a honeycomb lattice.

PRL 104, 187203 (2010) PHYSICAL REVIEW LETTERS week ending 7 MAY 2010

Frustrated Antiferromagnets with Entanglement Renormalization: Ground State of the Spin-$\frac{1}{2}$ Heisenberg Model on a Kagome Lattice

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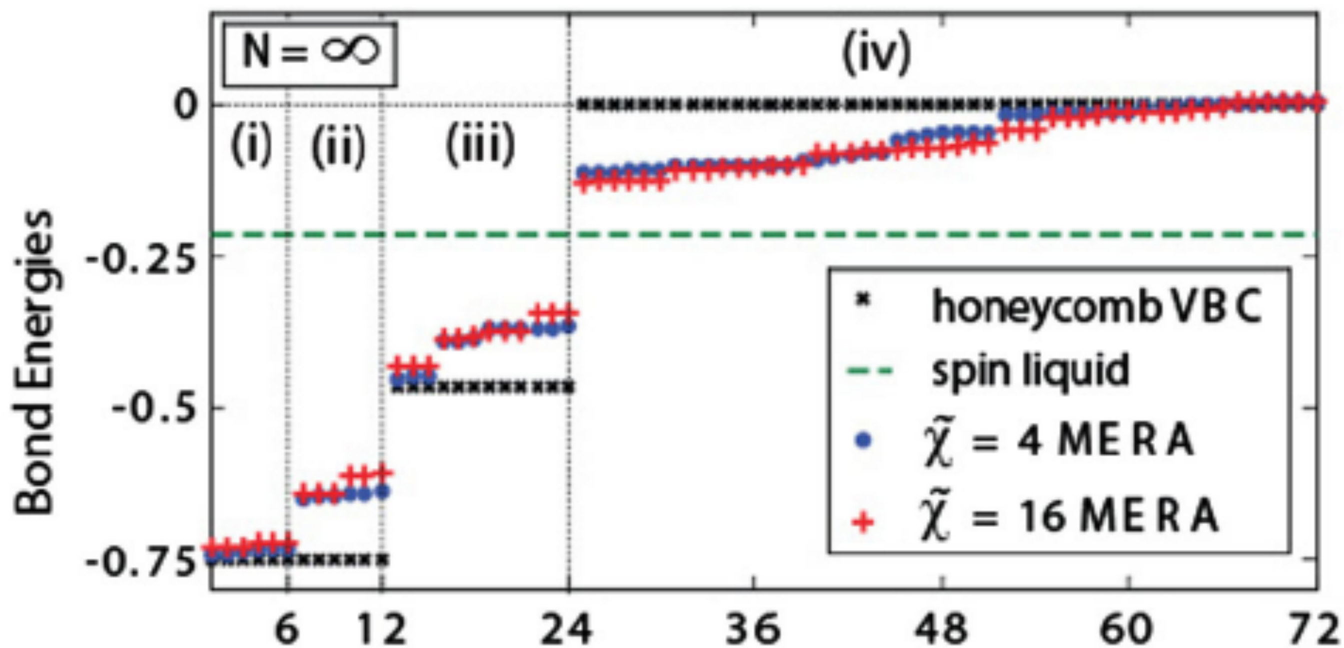
(Received 23 April 2009; revised manuscript received 10 December 2009; published 7 May 2010)

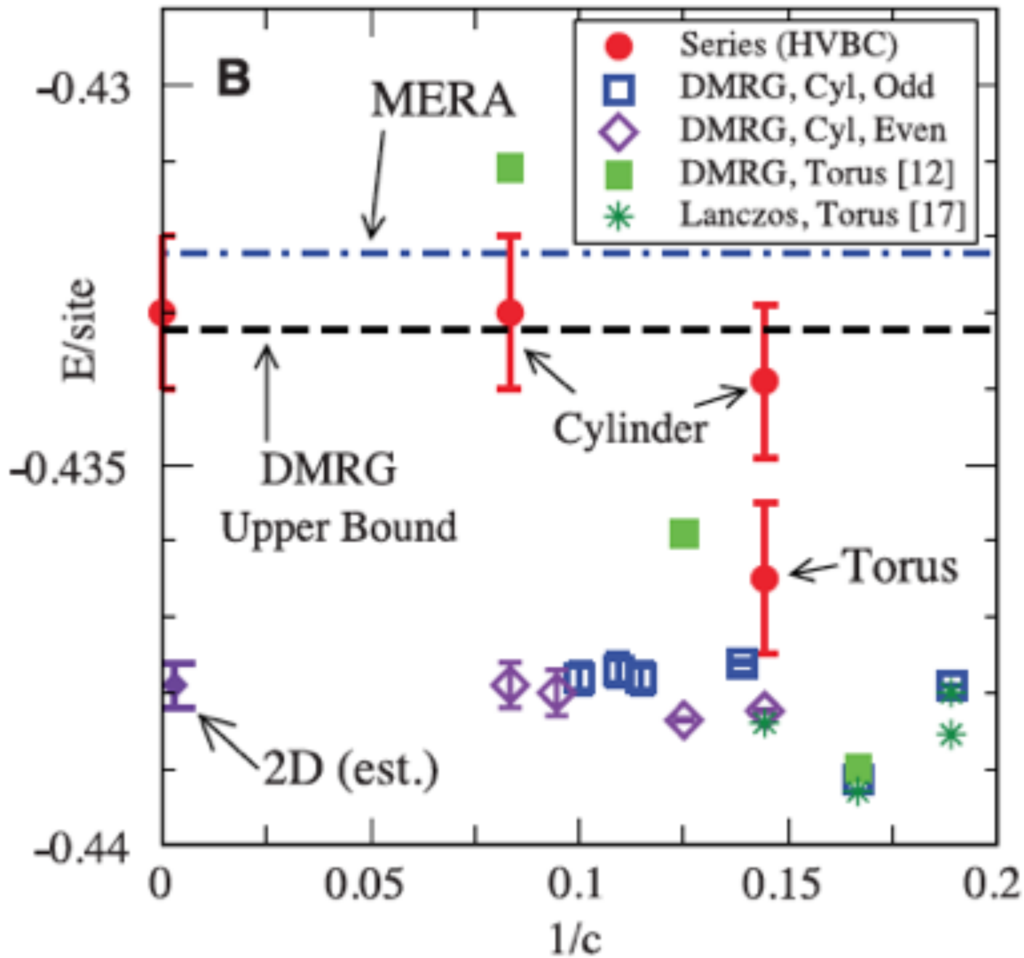
Multiscale Entanglement Renormalization Ansatz (MERA)



Spin-Liquid Ground State of the $S = 1/2$ Kagome Heisenberg Antiferromagnet

Simeng Yan,[1] David A. Huse,[2,3] Steven R. White[1]*

DMRG for cylinder up to circumference of 12



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Simeng Yan,[1] David A. Huse,[2,3] Steven R. White[1*]

SCIENCE VOL 332 3 JUNE 2011

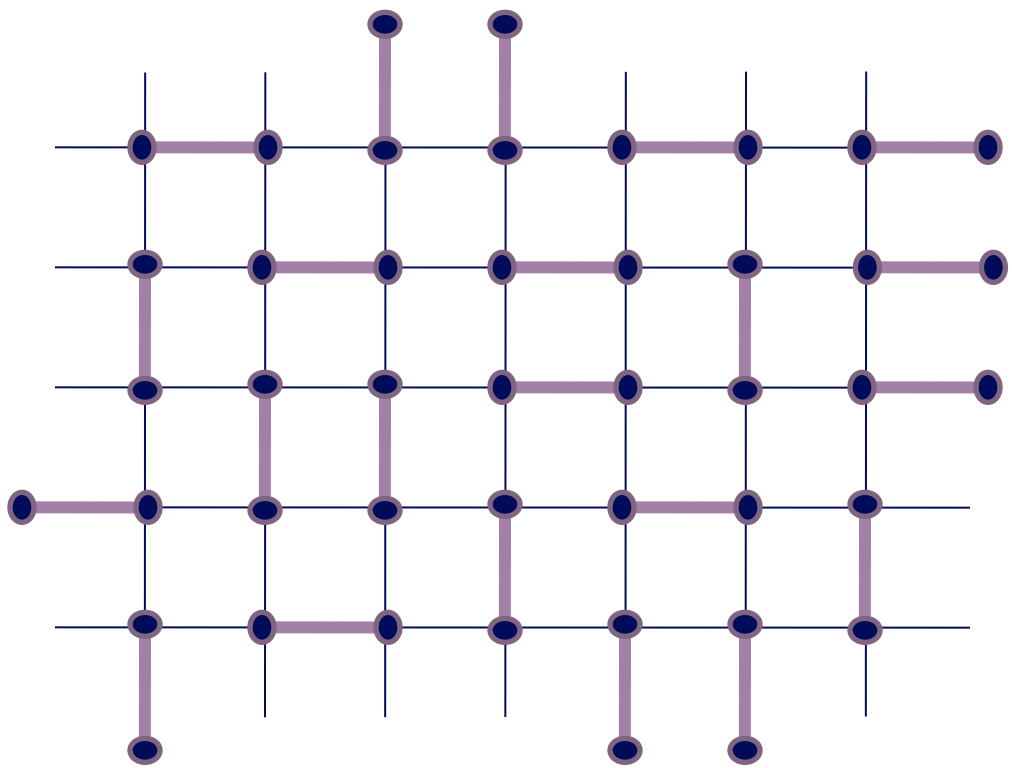
The ground state that we find has only short-range correlations, with a nonzero energy gap for any excitations, including spin-singlet excitations. We have tested the response of this ground state to many sorts of perturbations that would select out ordered states, if they exist, without detecting any signs of any ordering. Thus, we conclude that the ground state is a gapped spin liquid.

RVB spin liquid?

Resonating-Valence-Bond (RVB) state: disordered groundstate without Neel order?

P. W. Anderson (1973, 1987)

$$\sum_{\text{dimer coverings}}$$



History of RVB spin liquid phase

Square lattice Heisenberg: no RVB (Neel order)

[various authors, - 1989]

Square lattice Quantum Dimer Model:
critical pt. between 2 VBC phases

[Rokhsar-Kivelson 1988]

Triangular lattice QDM:
numerical evidence for gapped RVB

[Moessner-Sondhi 2001]

History of RVB spin liquid phase II

Exactly solvable models of gapped RVB phase
= "Z_2 topological phase"

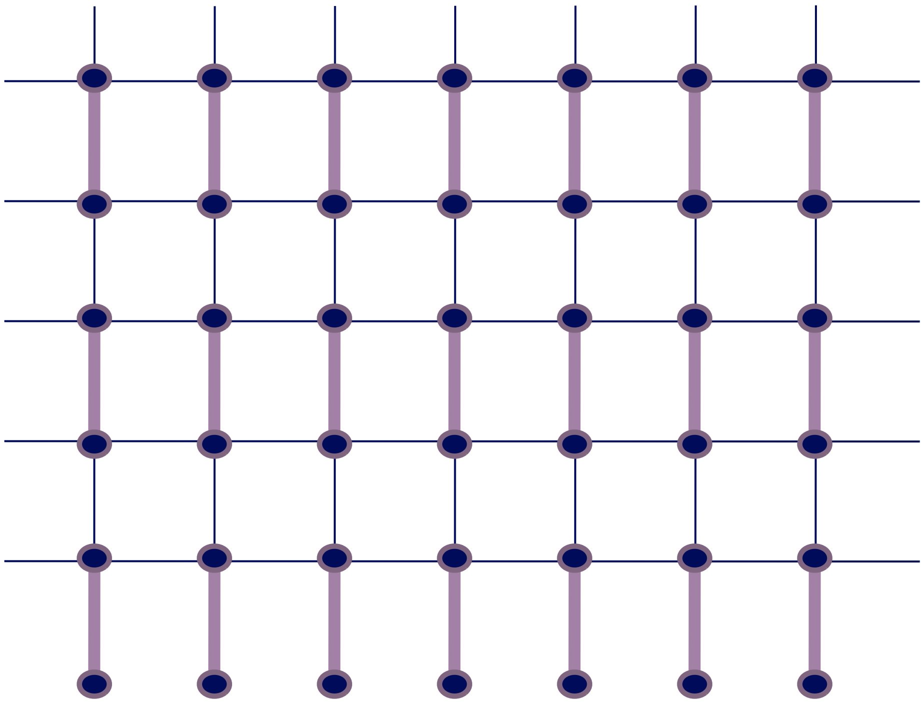
"Toric code" [A. Kitaev (1997)]

Kagome lattice QDM

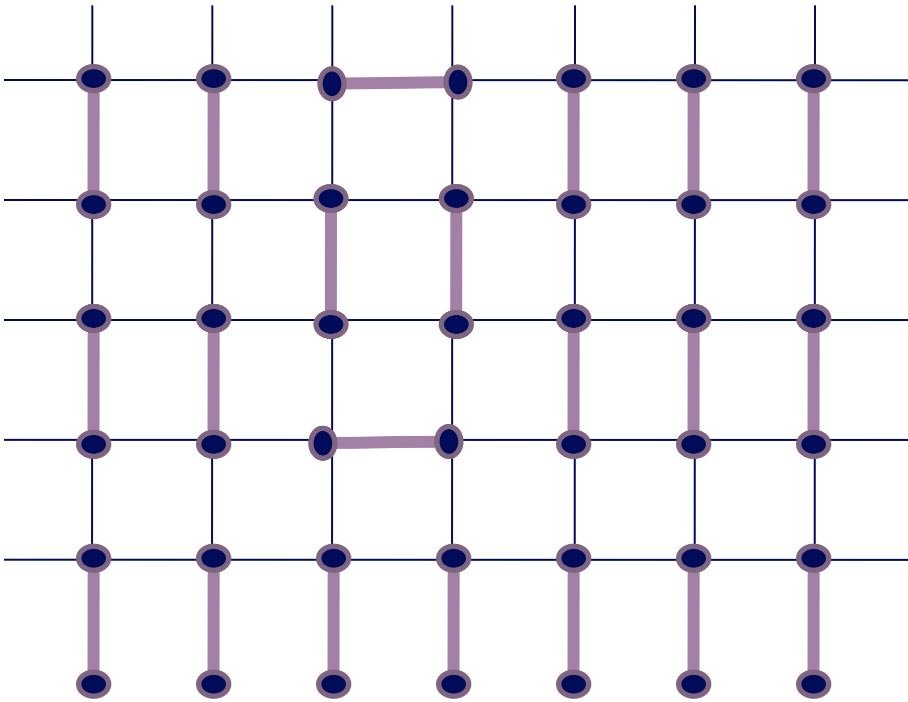
[Misguich-Serban-Pasquier (2002)]

RVB and "loop gas"

Reference state = Valence Bond Crystal (VBC)



Reference

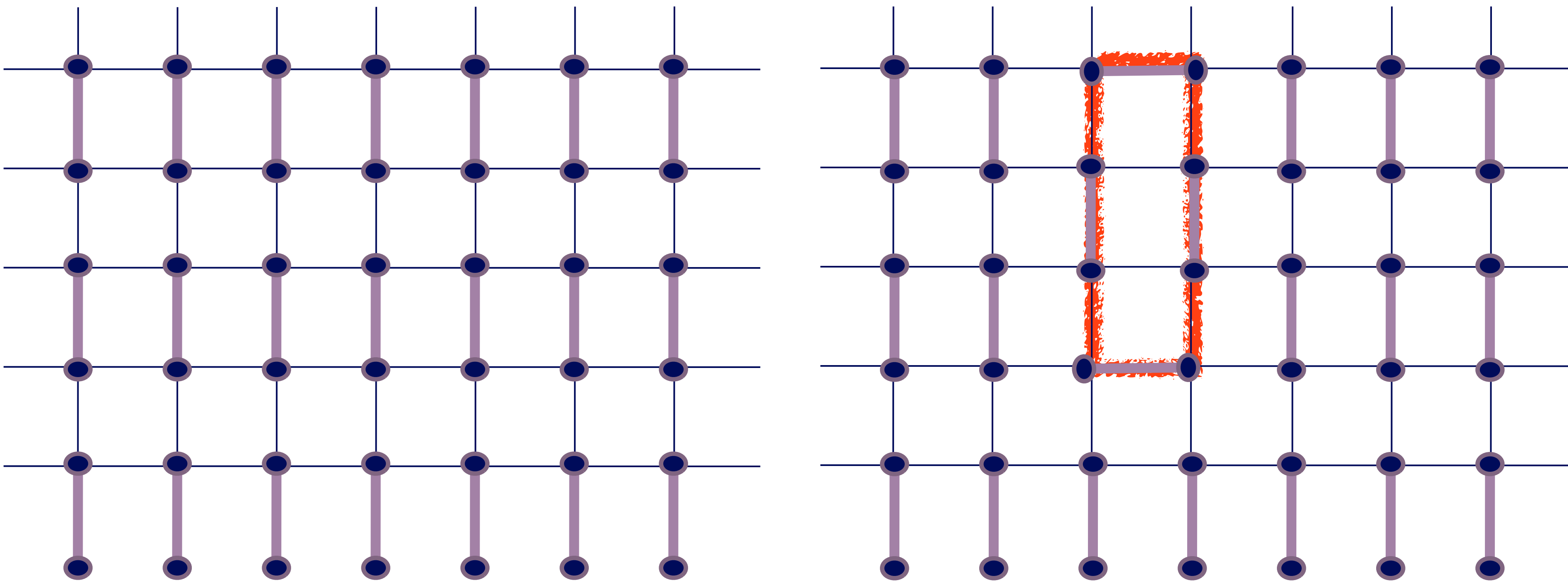


"new" state

RVB and “loop gas”

Reference state = Valence Bond Crystal (VBC)

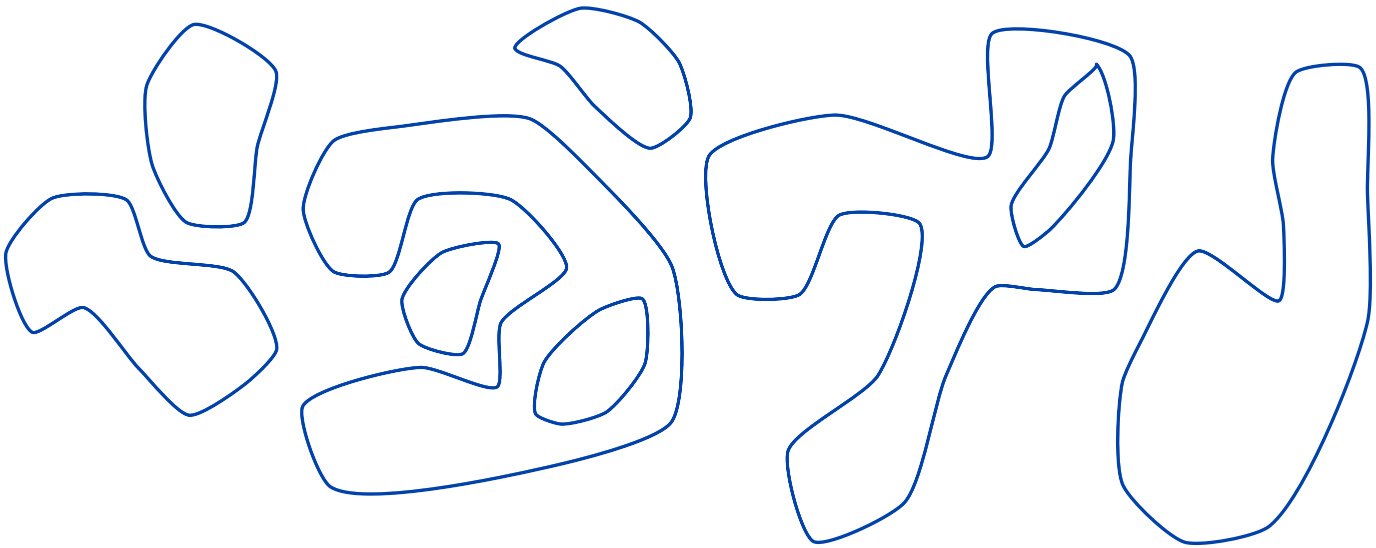
“resonance loop”



Reference “new” state

Z_2 topological phase

is given by a quantum superposition of configurations with closed loops



Proliferation (condensation) of loops
$\Leftrightarrow$ Z_2 topological phase

Non-condensation of loops:

VBC order is preserved (conventional order)

Condensation (proliferation) of loops:

Destruction of VBC order, without any conventional (magnetic, etc.) orders

= RVB phase

Problem:

There are now many (artificial) models which are known to belong to topological phases

How we can realize these phases, such as "spin liquid phase" in experiment, or in realistic models? $\Rightarrow$

S=1/2 HAF on kagome lattice?

Even if it is the case, how can we verify the existence of the topological phase, experimentally or numerically?

Identifying topological phases

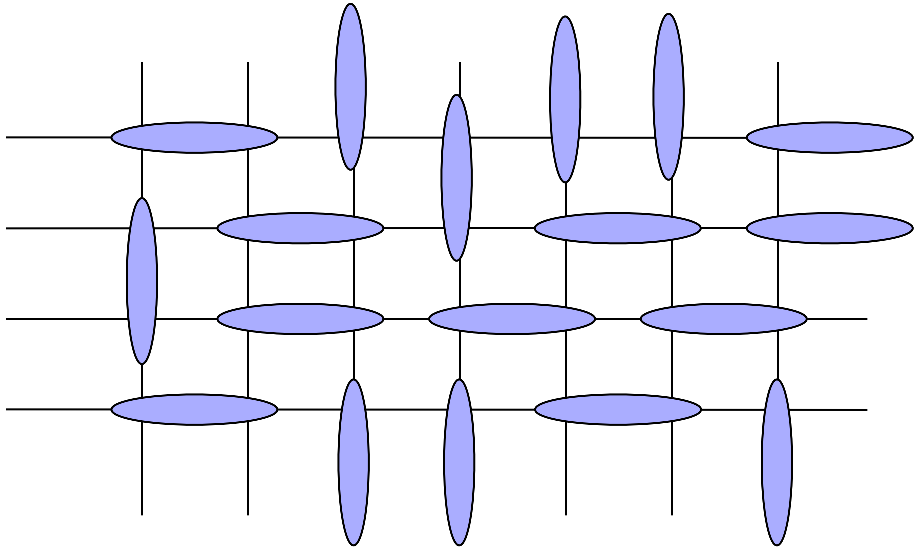
Experimental/numerical verification of "spin liquid" usually starts with showing *absence of any long-range order (magnetic, dimer, etc.)*

= **"negative" evidence**

"positive" experimental evidences may be obtained from excitations with fractional charge and/or statistics, etc. but more complicated

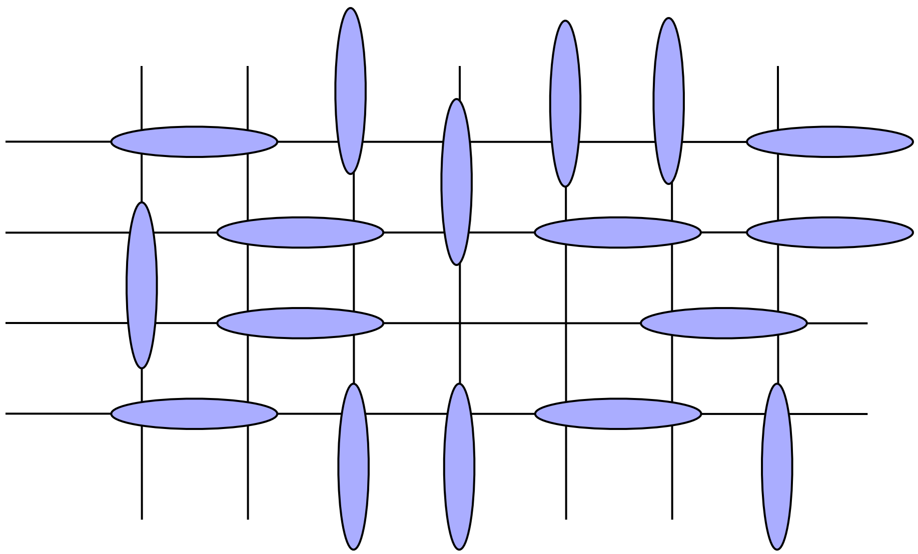
It has been also difficult to obtain positive evidences in numerics

Fractionalization in RVB liquid



Removing one dimer (i.e. doping a “hole”) can create two “half-dimer” hole excitations (spinons = fractionalized particle)

Fractionalization in RVB liquid



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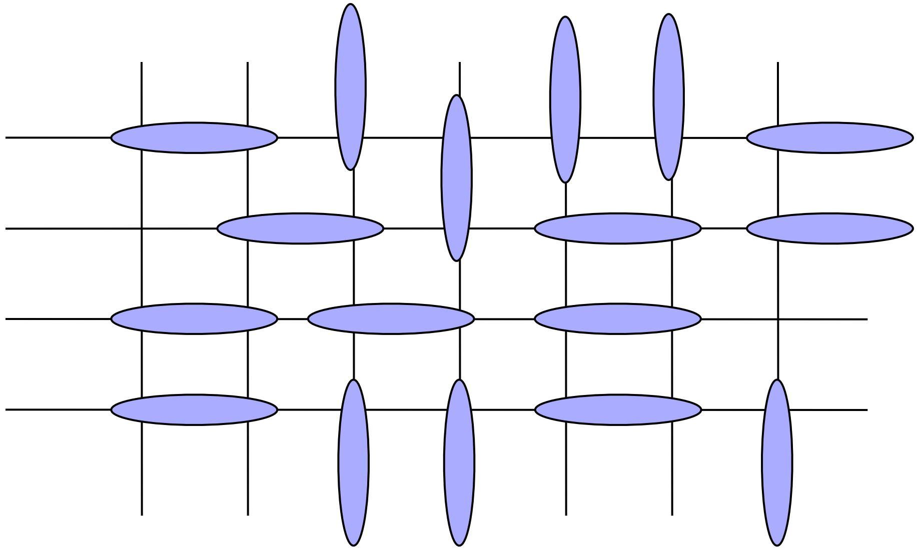
Fractionalization in RVB liquid



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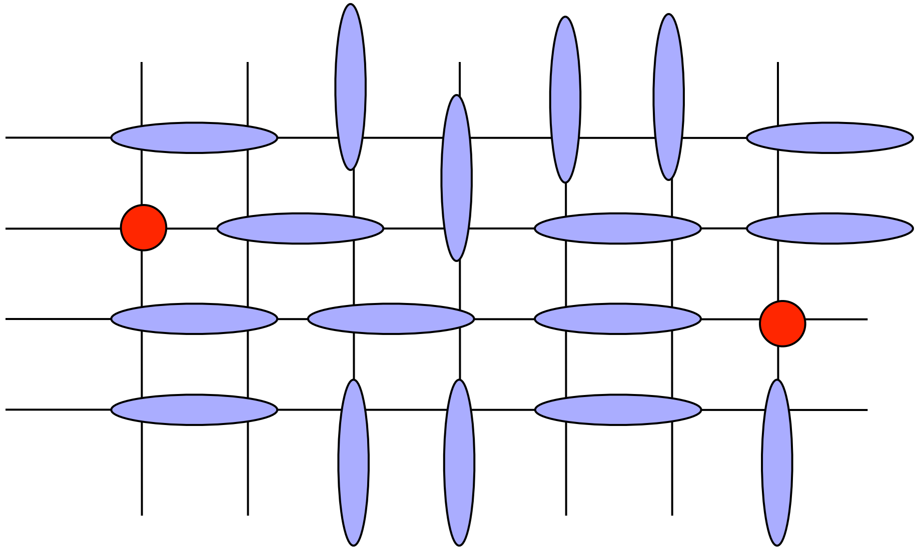


Removing one dimer (i.e. doping a "hole") can create two "half-dimer" hole excitations (spinons = fractionalized particle)

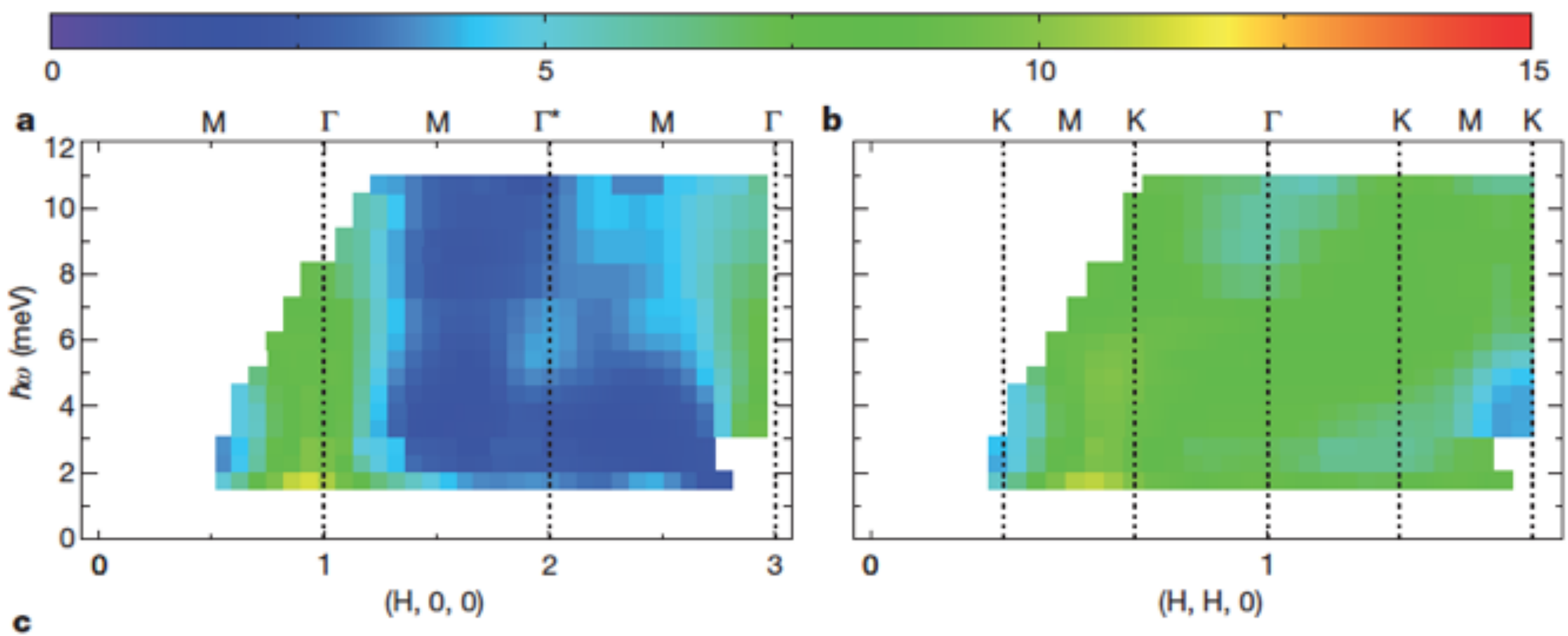
LETTER

[herbertsmithite]

doi:10.1038/nature11659

Fractionalized excitations in the spin-liquid state of a kagome-lattice antiferromagnet

Tian-Heng Han[1], Joel S. Helton[2], Shaoyan Chu[3], Daniel G. Nocera[4], Jose A. Rodriguez-Rivera[2,5], Collin Broholm[2,6] & Young S. Lee[1]



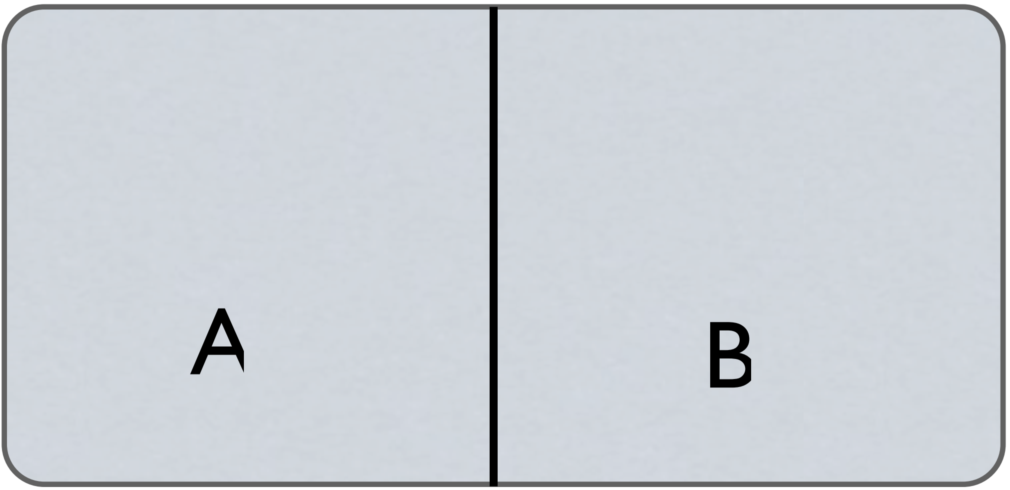
Recent neutron scattering expt. on herbertsmithite (T. H. Han, et al.)

✔ fractionalized spinon excitation

gapless excitation?
(inconsistent with the Z_2 spin liquid,
maybe the gap is too small....)

Topological Entanglement Entropy

Kitaev-Preskill, Levin-Wen 2006



$$\rho_A = \mathrm{Tr}_B |\Psi\rangle\langle\Psi|$$

$$S_E = -\mathrm{Tr}\left[\rho_A \log \rho_A\right]$$

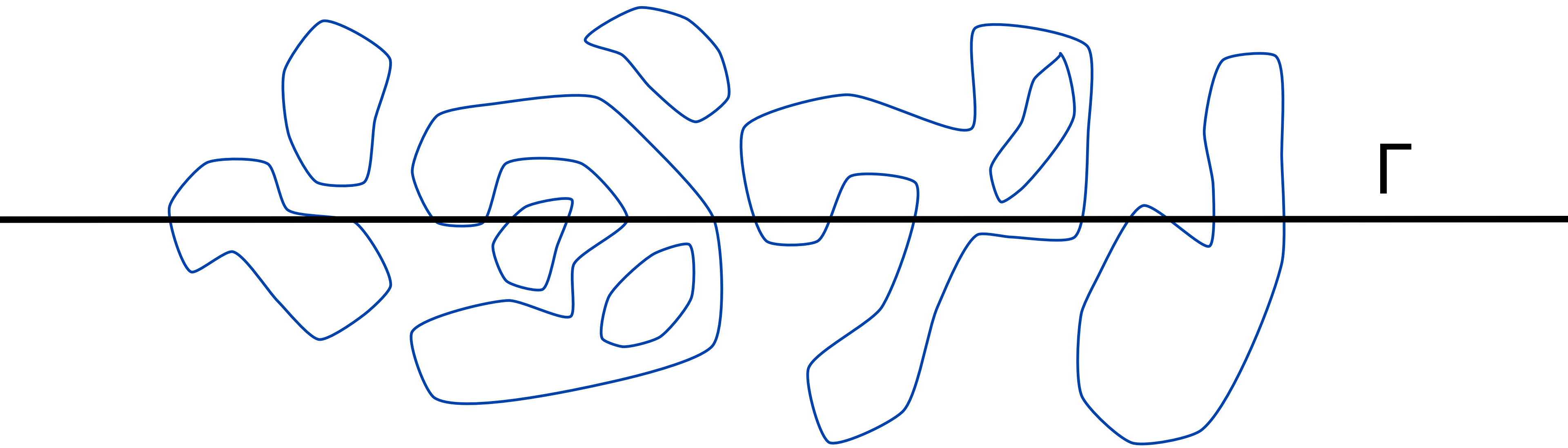
$$S_E \sim \alpha \mathfrak{L} - \gamma_{topo}$$

$\mathfrak{L}$: Boundary length

universal TEE

“area law” term
(non-universal coefficient α)

TEE in Z2 topological phase



Each boundary link may be crossed (q_l=1) or not (q_l=0) $N_q = 2^{\mathfrak{L}}$?

In fact, the number of string crossings at the boundary must be even, since the strings form closed loops $N_q = 2^{\mathfrak{L}-1}$

$$S_E = \log N_q = \mathfrak{L}\log 2 - \log 2$$

“area law” TEE

TEE is a ground-state property

No need to look at excitations!

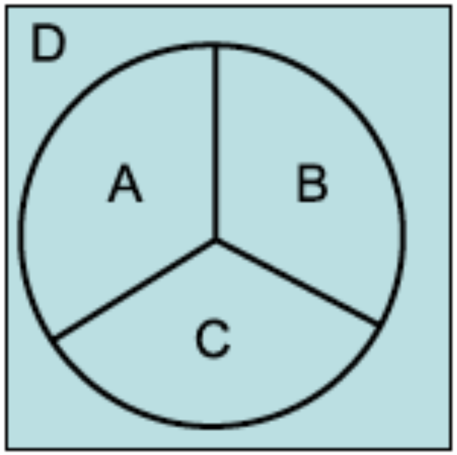
Measuring TEE in experiments is unrealistic, but for numerics it looks a convenient quantity characterizing topological phases

However, in numerics, we need to extract the subleading constant term, which is TEE

$$S_E \sim \alpha \mathfrak{L} - \gamma_{topo}$$

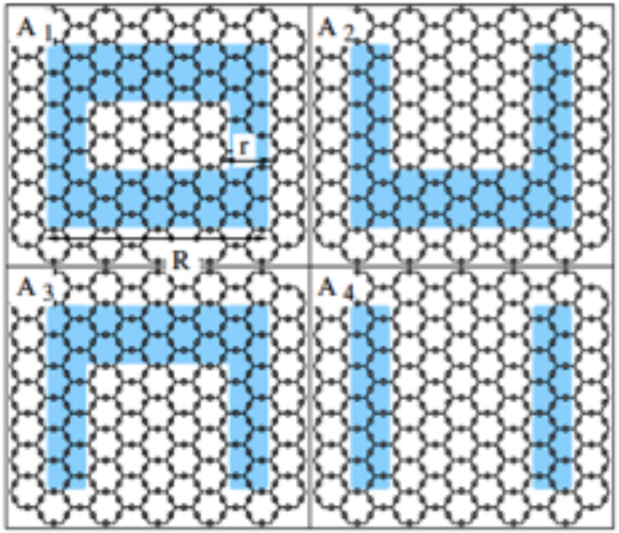
How to extract?

Kitaev-Preskill



$$S_{\rm topo} \equiv S_A + S_B + S_C - S_{AB} - S_{BC} - S_{AC} + S_{ABC}.$$

Levin-Wen



$$(S_1 - S_2) - (S_3 - S_4)$$

Cancellation of “area law” term

→ extraction of universal constant (TEE)

However, in practical applications, large finite-size effect due to curvature can be problematic

cf.) curvature expansion [Grover-Turner-Vishwanath 2011]

Simpler approach

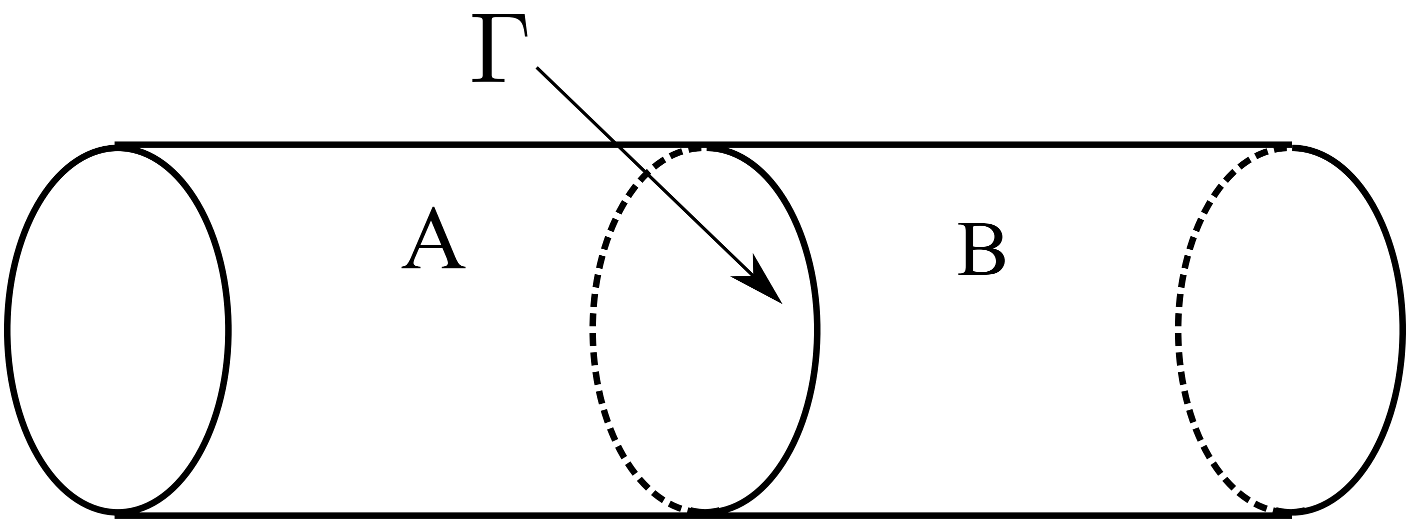
$$S_E \sim \alpha \mathfrak{L} - \gamma_{topo}$$

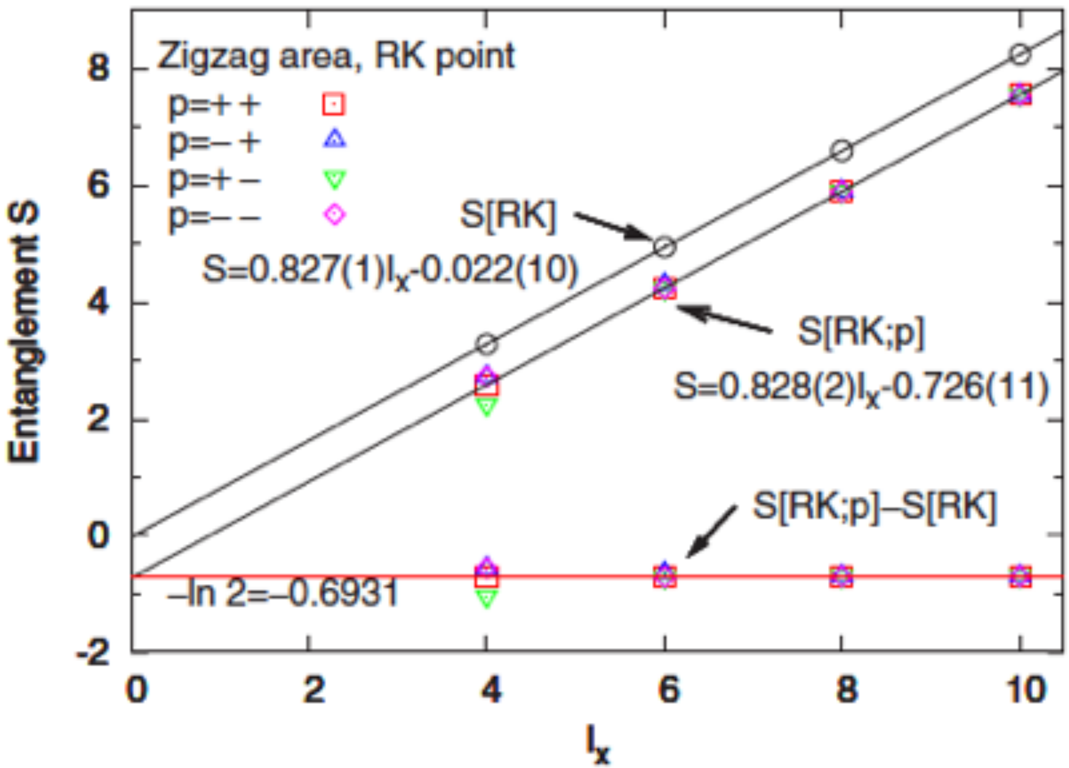
Measure EE for various boundary lengths, then "extrapolate" to zero length

"Smooth" boundary gives less finite-size effects

Problem: generally, there is an ambiguity in defining the boundary length - which leads to ambiguity in estimate of TEE

Solution:



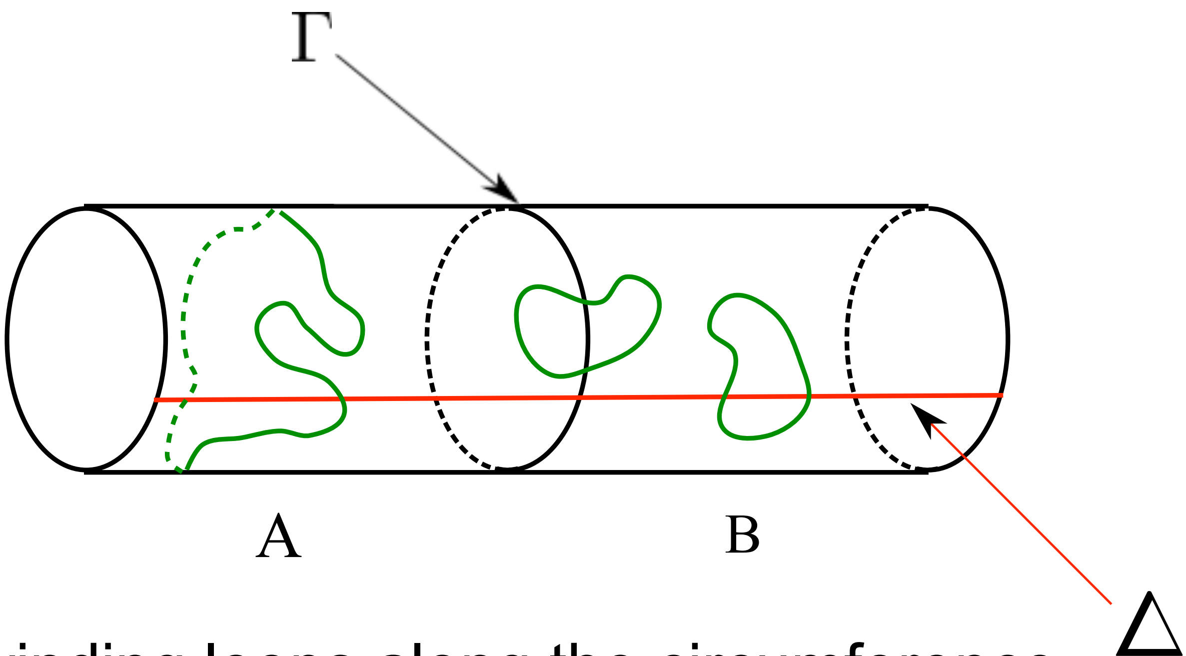


Quantum Dimer Model on triangular lattice (Z_2 topological phase)

extrapolation works quite well

[Furukawa-Misguich 2007]

Z_2 topological phase on cylinder



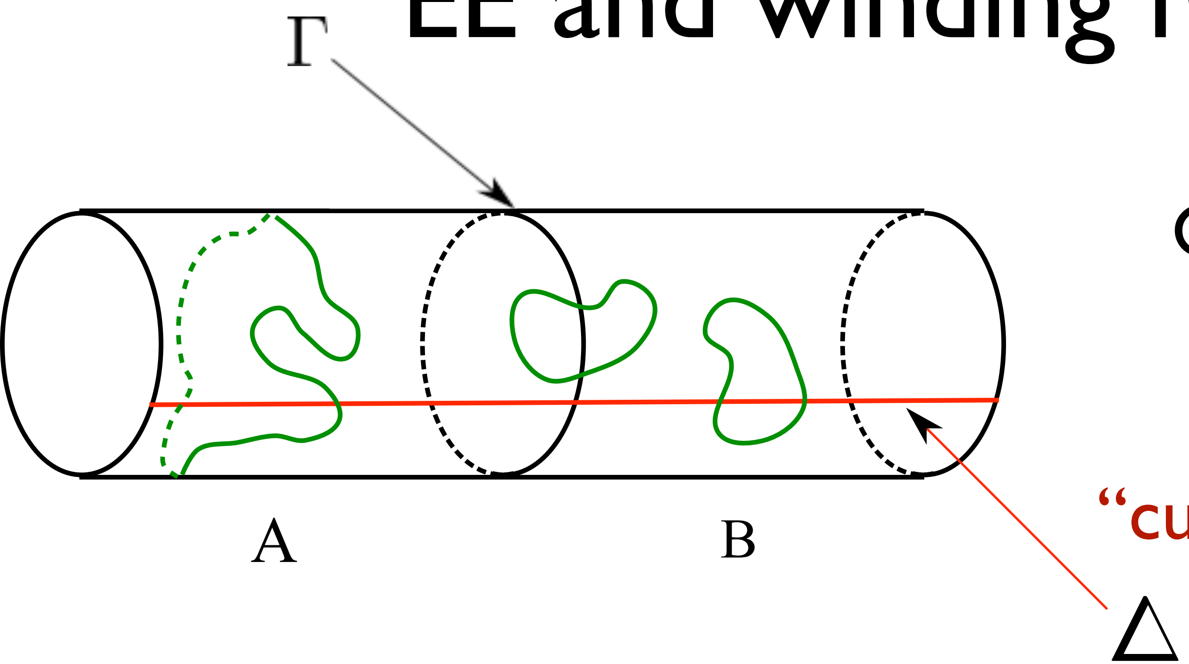
Strings can form winding loops along the circumference

Winding number is conserved modulo 2

$\Rightarrow$ doubly degenerate GS

$|\xi_{0,1}\rangle$ winding number = 0,1 (modulo 2)

EE and winding number



Consider one of the "pure" GS $|\xi_{0,1}\rangle$

Crossing # at the cut = winding # (mod 2)

$$|\xi_1\rangle = \frac{1}{\sqrt{2N_q}} \sum_{\{q_l\}} \left(|\Psi^A_{\{q_l\},0}\rangle |\Psi^B_{\{q_l\},1}\rangle + |\Psi^A_{\{q_l\},1}\rangle |\Psi^B_{\{q_l\},0}\rangle \right)$$

crossing # at the cut (mod 2) within A or B

$$S_E = \mathfrak{L} \log 2$$

"Topological EE" vanishes, owing to the extra entanglement

$$|\Psi\rangle = c_0|\xi_0\rangle + c_1|\xi_1\rangle$$

"TEE"

$$S_E = \mathfrak{L}\log 2 - [\log 2 - S_{cl}(\{\tilde{p}_0, \tilde{p}_1\})]$$

$$\tilde{p}_0 = \frac{|c_0 + c_1|^2}{2}$$

$$\tilde{p}_1 = \frac{|c_0 - c_1|^2}{2}$$

$$S_{cl}(\{p_\mu\}) = -\sum_\mu p_\mu \log p_\mu$$

TEE depends on the groundstate, in the presence of topological degeneracy!

Hamma-Ioniciou-Zanardi 2005, Furukawa-Misguich 2007
Dong-Fradkin-Leigh-Nowling 2008
Zhang-Grover-Turner-MO-Vishwanath 2011

Useful information on quasiparticle statistics can be obtained from this dependence

Minimal Entropy States (MES)

Zhang-Grover-Turner-MO-Vishwanath 2011

Among the topologically degenerate groundstates, there are “minimal entropy states” with maximal TEE

$$S_E = \mathfrak{L}\log 2 - [\log 2 - \cancelto{0}{S_{cl}(\{\tilde{p}_0, \tilde{p}_1\})}]$$

In the MES, the universal TEE of log 2 (for Z_2 topological phase) is recovered

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DMRG for cylinder up to circumference of 12

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Positive evidence for TP?

Problems:

1) **No clear evidence of topological degeneracy** on cylinder, from DMRG

2) TEE may be used as an evidence of Z2 topological phase - but what to do about the **groundstate dependence of TEE**??

These may be solved naturally

Jiang-Wang-Balents, Nature Physics 2012

DMRG gives γ_{topo}=log 2

Kagome w/ NNN exchange J_2 **[Jiang-Wang-Balents 2012]**

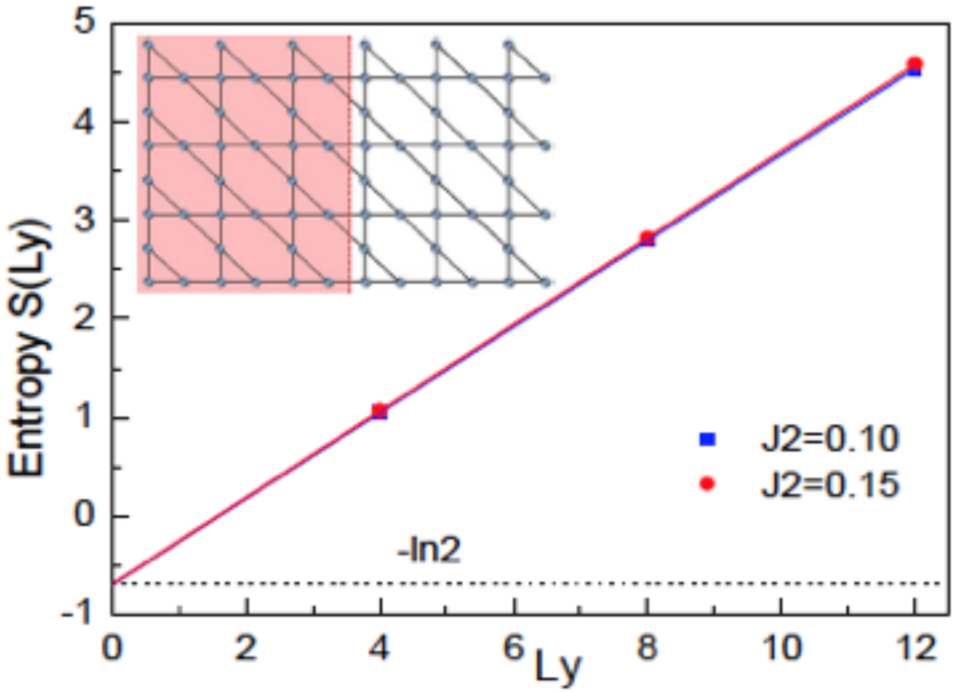


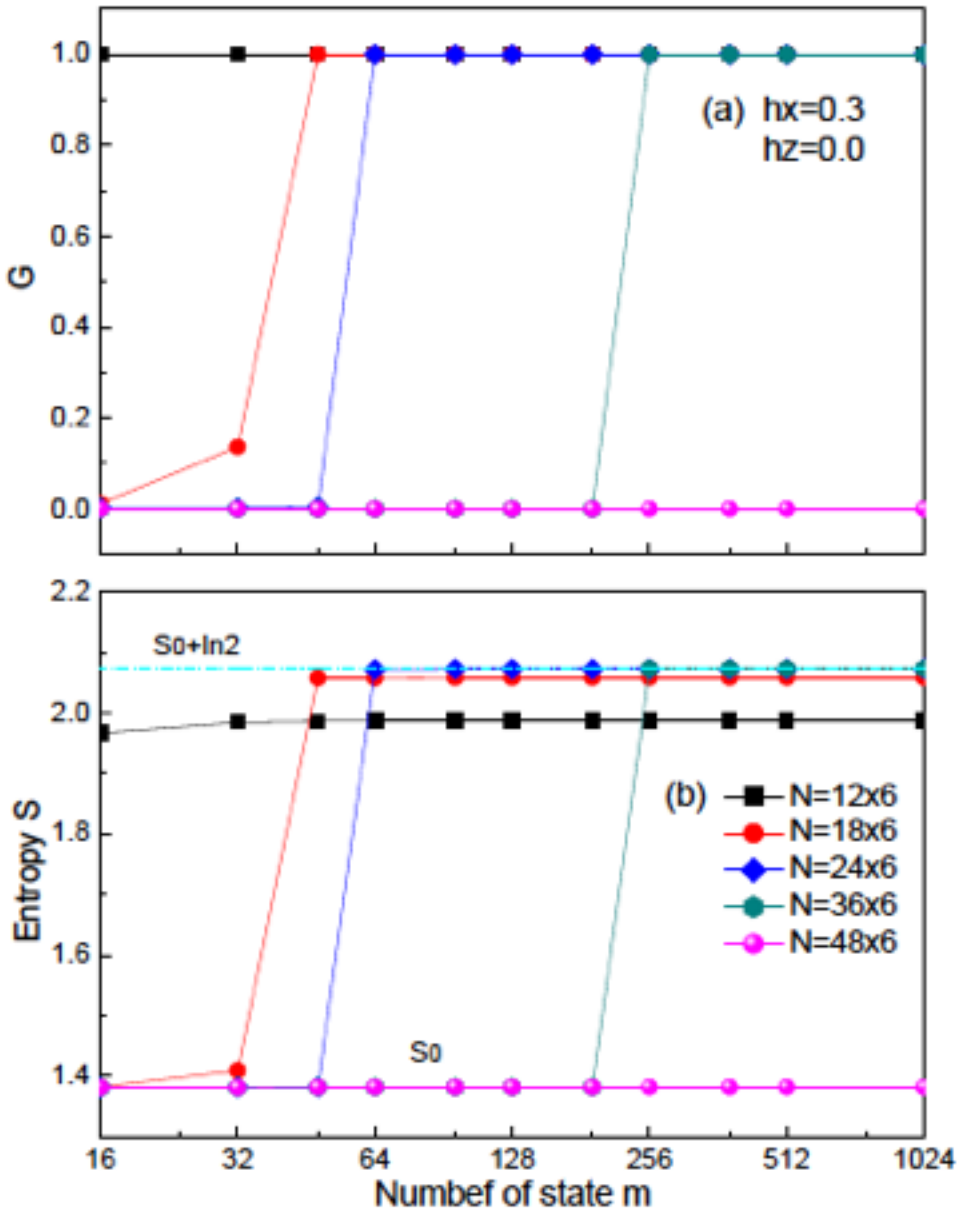
Figure 3: The entanglement entropy $S(L_y)$ of the kagomé J_1-J_2 model in Eq.(2), with $L_y = 4 \sim 12$ at $L_x = \infty$. By fitting $S(L_y) = aL_y - \gamma$, we get $\gamma = 0.698(8)$ at $J_2 = 0.10$, and $\gamma = 0.694(6)$ at $J_2 = 0.15$. Inset: kagomé lattice with $L_x = 12$ and $L_y = 8$.

Why?

Why the "ideal" value of TEE was obtained, without selecting the MES intentionally?

Answer:
DMRG naturally favors MES, and groundstates other than MES requires exponentially large number of states kept "*m*" and cannot be realistically seen in larger systems **[Jiang-Wang-Balents 2012]**

This also explains the lack of evidences for topological degeneracy from DMRG



demonstration
for toric code
in magnetic field

[Jiang-Wang-Balents 2012]

DMRG on "pure" kagome (without J2) also shows TEE consistent with Z2 TP

[Depenbrock-McCulloch-Schollwöck 2012]

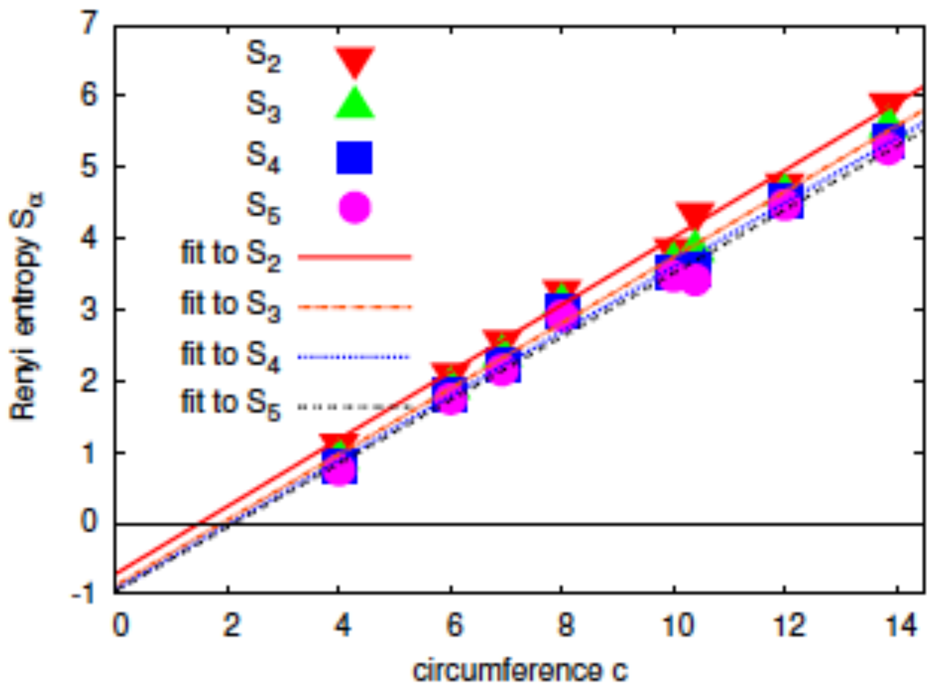


FIG. 6. Renyi entropies S_α of infinitely long cylinders for various α versus circumference c, extrapolated to $c = 0$. The negative intercept is the topological entanglement entropy γ.

“*F*-theorem” in 2+1D CFT

Entanglement entropy across a circle

$$S_E \sim \alpha \mathcal{L} - \gamma$$

coincides with the “topological EE” for topological phases, but can be also defined for critical points (CFT)

Casini-Huerta-Myers: γ is also the universal part of the free energy of the CFT on 3-sphere

Myers-Sinha: γ decreases monotonically along RG!

Application of *F*-theorem

T. Grover: **arXiv:1211.1392**

Free boson (scalar): $\gamma \sim 0.0638$

O(3) Gaussian critical point: $\gamma_{O(3)} < 0.0638\times 3$

Ising critical point: $\gamma_{\mathrm{Ising}} < 0.0638$

Z_2 topological phase: $\gamma = \log(2) = 0.693$

There cannot be an RG flow from O(3) Gaussian or Ising critical point to the Z_2 topological phase!

(explains why it is difficult to realize the topological phase?)

Conclusion

"(Gapped) RVB spin liquid phase" had been elusive for long time. But its existence is now well established theoretically, at least in special models. It is "Z_2 topological phase" in modern terminology.

The standard S=1/2 Heisenberg AFM on kagome lattice may realize the RVB spin liquid phase - positive numerical evidence from Topological Entanglement Entropy.